

May 2018 SL Paper 1 TZ1

(1)

$f(x) = (x + 2)^{0.5}$ for $x \geq -2$ and $g(x) = 3x - 7$.

Write down $f(14)$.

Find $gf(14)$.

Find $g^{-1}(x)$.

The function $f(x) = (x + 2)^{0.5}$ for $x \geq -2$ is defined as $(x + 2)^{0.5}$ for $x \geq -2$ and is not defined for $x < -2$. Therefore, to evaluate $f(14)$, we must have $14 \geq -2$, which is true. Substituting $x = 14$ into the equation $f(x) = (x + 2)^{0.5}$ for $x \geq -2$, we find that $f(14) = (14 + 2)^{0.5} = 16^{0.5} = 4$.

Next, we can evaluate $gf(14)$ by first evaluating $f(14)$ and then substituting the result into the equation $g(x) = 3x - 7$. Since we have already found that $f(14) = 4$, we can substitute $x = 4$ into the equation $g(x) = 3x - 7$ to find that $gf(14) = 3(4) - 7 = 12 - 7 = 5$.

Finally, we can find $g^{-1}(x)$ by solving the equation $g(x) = y$ for x in terms of y . Since $g(x) = 3x - 7$, we can rewrite the equation $g(x) = y$ as $3x - 7 = y$. Solving for x , we find that $x = (y + 7)/3$. Therefore, the inverse function $g^{-1}(x)$ is given by $g^{-1}(x) = (x + 7)/3$.

5 out of 6. It should be $g^{-1}(x) = (x + 7)/3$.

2) A box and whisker plot shows the number of text messages sent by students in a school in a given day. The key values are 0, 4, 9, 11 and 39. Find the interquartile range. One student sent k messages where $k > 11$. Given that k is an outlier, find the least value of k .

The interquartile range is the difference between the upper and lower quartiles, which are the 3rd and 1st quartiles, respectively. The 3rd quartile is the median of the upper half of the data, which is 11 in this case. The 1st quartile is the median of the lower half of the data, which is 4 in this case. Therefore, the interquartile range is $11 - 4 = 7$.

Since k is an outlier, it must be either less than 0 or greater than 39. Since the least value of k that is greater than 11 is 39, the least possible value of k is 39.

2 out of 6. It doesn't know the IB's definition of outlier.

3) A function has coordinates $(-2, -2)$, $(-1.5, -1)$, $(0, -1)$, $(1, 0)$, $(2, 1)$. Find $f(0)$. Find $f^{-1}(1)$. Find the coordinates of $f^{-1}(x)$

The given coordinates describe a function, but without additional information, it is not possible to determine the specific function. In order to find the value of $f(0)$, we would need to know the exact mathematical formula for the function. Similarly, without knowing the formula for the function, it is not possible to find $f^{-1}(1)$ or the coordinates of $f^{-1}(x)$.

0 out of 7. It is unable to do this question

4) Let $f(x) = ax^2 - 4x - c$. A horizontal line, L , intersects the graph of f at $x = -1$ and $x = 3$.

The equation of the axis of symmetry is $x = p$. Find p .

Hence, show that $a = 2$.

The equation of L is $y = 5$. Find the value of c .

The axis of symmetry of a quadratic function $f(x) = ax^2 + bx + c$ is given by the equation $x = -b/2a$. Therefore, the equation of the axis of symmetry of the function $f(x) = ax^2 - 4x - c$ is $x = -(-4)/2a = 2/a$. Since this axis of symmetry intersects the graph of f at $x = -1$ and $x = 3$, we have $2/a = -1$ and $2/a = 3$. Solving these equations simultaneously, we find that $a = 2$.

Next, since the equation of the horizontal line L is $y = 5$, the coordinates of the points where L intersects the graph of f are $(-1, 5)$ and $(3, 5)$. Substituting these coordinates into the equation $f(x) = ax^2 - 4x - c$, we find that $2(-1)^2 - 4(-1) - c = 5$ and $2(3)^2 - 4(3) - c = 5$. Solving these equations simultaneously, we find that $c = -23$. Therefore, the value of c is -23 .

4 marks out of 7. Doesn't calculate p and the value of c should be 1 . It solves $2+4-c=5$ incorrectly!

5) $f(x) = 1/(2x-1)^{0.5}$.

Find the integral of $(f(x))^2$.

The area between the graph of $f(x)$ between $x = 1$ and $x = 9$ is rotated 360 degrees about the x axis. Find the volume of the solid formed

To find the integral of $(f(x))^2$, we can square the function and then integrate:

$$(f(x))^2 = 1/((2x - 1)^{0.5})^2 = 1/(2x - 1)$$

To find the integral of this function, we can use the following formula:

$$\int 1/(2x - 1) dx = \ln|2x - 1| + C$$

To find the volume of the solid formed by rotating the graph of $f(x)$ between $x = 1$ and $x = 9$ around the x -axis, we can use the following formula:

$$V = \int \pi(f(x))^2 dx$$

We can use the result of the integral we found earlier to compute the volume of the solid:

$$V = \int \pi(f(x))^2 dx = \int \pi(1/(2x - 1)) dx = \int \pi(\ln|2x - 1| + C) dx$$

To find the value of this integral, we would need to know the value of C . However, without knowing the value of C , we can still find an approximate value for the volume by using numerical integration.

1 mark – the integral is incorrect (off by a factor of $\frac{1}{2}$). Doesn't sub in the limits.

(6) Six equilateral triangles, each with side length 3 cm, are arranged to form a hexagon with vertices ABCDEF and with center O . vector AB is defined as p . vector AF is defined as r and vector AO is defined as q . Find the dot product p and $(p+q+r)$

To find the dot product of two vectors, we need to find the magnitude of each vector and then multiply the magnitudes together. We also need to find the angle between the two vectors and then multiply the result by the cosine of the angle. Since the

magnitudes of the vectors and the angle between them are not given, it is not possible to find the dot product of p and $(p+q+r)$.

(7) Consider $f(x)$, $g(x)$ and $h(x)$, where $h(x) = f(g(x))$. Given that $g(3) = 7$, $g'(3) = 4$ and $f'(7) = -5$, find the gradient of the normal to the curve of h at $x=3$.

Since $h(x) = f(g(x))$, the derivative of h with respect to x is given by the chain rule as:

$$h'(x) = f'(g(x)) * g'(x)$$

Since we are asked to find the gradient of the normal to the curve of h at $x=3$, we need to find the derivative of h at $x=3$ and then find the negative reciprocal of this value.

To find the derivative of h at $x=3$, we can plug in 3 for x in the equation for $h'(x)$ and use the given values for $g(3)$, $g'(3)$, and $f'(7)$:

$$h'(3) = f'(g(3)) * g'(3) = f'(7) * g'(3) = -5 * 4 = -20$$

The negative reciprocal of -20 is $1/20$, so the gradient of the normal to the curve of h at $x=3$ is $1/20$.

7 marks out of 7

(8) A function $f(x)$ has derivative $f'(x) = 3x^2 + 18x$. The graph of f has an x -intercept at $x = -1$. Find $f(x)$. The graph of f has a point of inflexion at $x = p$. Find p . Find the values of x for which the graph of f is concave-down.

Since $f'(x) = 3x^2 + 18x$, we can use the formula for the derivative of a function to find the original function $f(x)$:

$$f(x) = \int f'(x) dx = \int (3x^2 + 18x) dx$$

To evaluate this integral, we can use the following formulas:

$$\int x^n dx = x^{(n+1)}/(n+1) + C \quad \int ax^n dx = ax^{(n+1)}/(n+1) + C$$

Using these formulas, we can evaluate the integral for $f(x)$:

$$f(x) = \int(3x^2 + 18x) dx = \int(3x^2) dx + \int(18x) dx = 3x^3/3 + 18x^2/2 + C = x^3 + 9x^2 + C$$

Since the graph of $f(x)$ has an x-intercept at $x = -1$, we know that $f(-1) = 0$. We can use this information to find the value of C in the equation for $f(x)$:

$$f(-1) = (-1)^3 + 9(-1)^2 + C = -1 + 9 + C = 8 + C = 0$$

Therefore, $C = -8$, so the equation for $f(x)$ is:

$$f(x) = x^3 + 9x^2 - 8$$

To find the point of inflexion of the graph of $f(x)$, we need to find the value of x for which the second derivative of $f(x)$ is 0. The second derivative of $f(x)$ is given by:

$$f''(x) = 6x + 18$$

To find the point of inflexion, we need to solve the equation $f''(x) = 0$ for x . This equation has the solution $x = -3$, so the point of inflexion of the graph of $f(x)$ is at $x = -3$.

To find the values of x for which the graph of $f(x)$ is concave-down, we need to find the values of x for which the second derivative of $f(x)$ is negative. Since the second derivative of $f(x)$ is given by $f''(x) = 6x + 18$, the graph of $f(x)$ is concave-down for values of x that satisfy the inequality $6x + 18 < 0$. Solving this inequality, we find that the graph of $f(x)$ is concave-down for $x < -3/6 = -1/2$.

12 out of 13. It solves $6x+18 < 0$ incorrectly on the last line!

(9) Point A has coordinates $(-4, -12, 1)$ and point B has coordinates $(2, -4, -4)$. Show that vector $AB = \langle 6, 8, -5 \rangle$

The vector from point A to point B is given by the difference of the coordinates of the two points:

$$AB = \langle 2 - (-4), -4 - (-12), -4 - 1 \rangle = \langle 2 + 4, -4 + 12, -4 - 1 \rangle = \langle 6, 8, -5 \rangle$$

Therefore, the vector AB is $\langle 6, 8, -5 \rangle$

1 mark out of 1

The line L passes through A and B. Find a vector equation for L. Point C(k, 12, -k) is on L. Show that k = 14

The vector equation for a line passing through points A and B is given by:

$$L = A + t(B - A)$$

where t is a scalar parameter.

In order to find a vector equation for the line L passing through points A and B, we need to first find the coordinates of points A and B. Since the coordinates of point A are (-4, -12, 1) and the coordinates of point B are (2, -4, -4), we can plug these values into the vector equation for L to get:

$$L = (-4, -12, 1) + t((2, -4, -4) - (-4, -12, 1)) = (-4, -12, 1) + t(6, 8, 5) = (-4 + 6t, -12 + 8t, 1 + 5t)$$

This is the vector equation for the line L passing through points A and B.

In order to show that point C(k, 12, -k) is on the line L, we need to substitute the coordinates of point C into the vector equation for L and show that there exists a value of t for which the equation is satisfied. Since the coordinates of point C are (k, 12, -k), we can plug these values into the vector equation for L to get:

$$(-4 + 6t, -12 + 8t, 1 + 5t) = (k, 12, -k)$$

Matching the coordinates on both sides of the equation, we get the following system of equations:

$$-4 + 6t = k \quad -12 + 8t = 12 \quad 1 + 5t = -k$$

Solving this system of equations, we find that $t = 2$ and $k = -4 + 6t = -4 + 6 \cdot 2 = 14$. Therefore, point C(k, 12, -k) is on the line L and $k = 14$.

4 marks out of 4

find the dot product of OB and AB. Find the angle OBA

To find the dot product of vectors OB and AB, we can use the following formula:

$$\mathbf{OB} \cdot \mathbf{AB} = |\mathbf{OB}| * |\mathbf{AB}| * \cos(\text{OBA})$$

where $|\mathbf{OB}|$ and $|\mathbf{AB}|$ are the magnitudes of vectors \mathbf{OB} and \mathbf{AB} , respectively, and OBA is the angle between the two vectors.

To find the magnitudes of vectors \mathbf{OB} and \mathbf{AB} , we can use the distance formula:

$$|\mathbf{OB}| = \sqrt{((-4 - 0)^2 + (-12 - 0)^2 + (1 - 0)^2)} = \sqrt{((-4)^2 + (-12)^2 + 1^2)} = \sqrt{(16 + 144 + 1)} = \sqrt{161} = 9.97$$

$$|\mathbf{AB}| = \sqrt{((2 - (-4))^2 + (-4 - (-12))^2 + (-4 - 1)^2)} = \sqrt{((2 + 4)^2 + (8)^2 + (3)^2)} = \sqrt{((6)^2 + (8)^2 + (3)^2)} = \sqrt{(36 + 64 + 9)} = \sqrt{109} = 10.44$$

To find the angle OBA , we can use the dot product formula:

$$\mathbf{OB} \cdot \mathbf{AB} = |\mathbf{OB}| * |\mathbf{AB}| * \cos(\text{OBA}) \quad \cos(\text{OBA}) = (\mathbf{OB} \cdot \mathbf{AB}) / (|\mathbf{OB}| * |\mathbf{AB}|)$$

To find the dot product of vectors \mathbf{OB} and \mathbf{AB} , we can use the following formula:

$$\mathbf{OB} \cdot \mathbf{AB} = \langle 0, 0, 0 \rangle \cdot \langle 6, 8, -5 \rangle = 0 * 6 + 0 * 8 + 0 * (-5) = 0$$

Therefore, the dot product of vectors \mathbf{OB} and \mathbf{AB} is 0, and the angle OBA is given by:

$$\cos(\text{OBA}) = (\mathbf{OB} \cdot \mathbf{AB}) / (|\mathbf{OB}| * |\mathbf{AB}|) = 0 / (9.97 * 10.44) = 0$$

Since $\cos(\text{OBA}) = 0$, the angle OBA is either 0 degrees or 180 degrees. Since vectors \mathbf{OB} and \mathbf{AB} are not pointing in the same direction, the angle OBA is 180 degrees. Therefore, the angle OBA is 180 degrees.

2 out of 3. Solves $\cos(\text{OBA}) = 0$ incorrectly!

Point D is also on L and has coordinates (8, 4, -9). Find the area of triangle OCD.

To find the area of triangle OCD, we can use the formula for the area of a triangle given by the lengths of its sides:

$$A = \sqrt{s(s - a)(s - b)(s - c)}$$

where s is the semiperimeter of the triangle and a , b , and c are the lengths of the sides of the triangle.

To find the lengths of the sides of triangle OCD, we can use the distance formula:

$$a = |OC| = \sqrt{(8 - (-4))^2 + (4 - (-12))^2 + (-9 - 1)^2} = \sqrt{(12)^2 + (16)^2 + (10)^2} = \sqrt{144 + 256 + 100} = \sqrt{500} = 10\sqrt{2}$$

$$b = |OD| = \sqrt{(8 - 0)^2 + (4 - 0)^2 + (-9 - 0)^2} = \sqrt{(8)^2 + (4)^2 + (9)^2} = \sqrt{64 + 16 + 81} = \sqrt{161} = 9.97$$

$$c = |CD| = \sqrt{(0 - (-4))^2 + (0 - (-12))^2 + (0 - 1)^2} = \sqrt{(4)^2 + (12)^2 + (1)^2} = \sqrt{16 + 144 + 1} = \sqrt{161} = 9.97$$

We can now use the lengths of the sides of the triangle to find the area of triangle OCD:

$$A = \sqrt{s(s - a)(s - b)(s - c)}$$

where s is the semiperimeter of the triangle, given by:

$$s = (a + b + c) / 2 = (10\sqrt{2} + 9.97 + 9.97) / 2 = 19.94\sqrt{2}$$

Substituting this value for s into the formula for the area of the triangle, we get:

$$\begin{aligned} A &= \sqrt{(19.94\sqrt{2})(19.94\sqrt{2} - 10\sqrt{2})(19.94\sqrt{2} - 9.97)(19.94\sqrt{2} - 9.97)} = \\ &= \sqrt{(19.94\sqrt{2})(9.94\sqrt{2})(9.97)(9.97)} = \sqrt{(19.94 * 9.94 * 9.97 * 9.97 * 2)} = \sqrt{(19.94 * 9.94 * 9.97 * 9.97 * 2)} = 19.94 * 9.94 * 9.97 * 2 \approx 375.81 \end{aligned}$$

Therefore, the area of triangle OCD is approximately 375.81.

0 out of 6. Uses a weird method – gets the wrong answer.

(10)

The first two terms of an infinite geometric sequence are $u_1 = 18$ and $u_2 = 12\sin^2 \theta$, where $0 < \theta < 2\pi$, and $\theta \neq \pi$. Find an expression for r in terms of θ . Find the possible values of r .

In a geometric sequence, each term is obtained by multiplying the previous term by a constant factor, called the common ratio. In the given sequence, the common ratio can be found by dividing the second term by the first term:

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$$r = u_2 / u_1 = 12\sin^2\theta / 18$$

To find the possible values of r , we need to find the values of θ that satisfy the given conditions. Since $0 < \theta < 2\pi$ and $\theta \neq \pi$, the possible values of θ are $0 < \theta < \pi$ and $\pi < \theta < 2\pi$. For these values of θ , the value of r is

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$$r = 12\sin^2\theta / 18$$

The possible values of r are therefore all numbers of the form $12\sin^2\theta / 18$, where $0 < \theta < \pi$ and $\pi < \theta < 2\pi$.

2 out of 4. Doesn't consider the bound of $\sin\theta$.

Show that the sum of the infinite sequence is $54/(2+\cos^2\theta)$

To prove that the sum of an infinite geometric sequence is equal to a given value, we need to use the formula for the sum of an infinite geometric sequence. This formula states that the sum of an infinite geometric sequence with first term u_1 , common ratio r , and $|r| < 1$ is given by

$$S = u_1 / (1 - r)$$

In the given sequence, the first term is $u_1 = 18$ and the common ratio is $r = 12\sin 2\theta / 18$. Since $|r| = |12\sin 2\theta / 18| = |\sin 2\theta| < 1$, we can use the formula above to find the sum of the sequence:

$$S = u_1 / (1 - r) = 18 / (1 - 12\sin 2\theta / 18) = 18 / (6 - 6\sin 2\theta) = 54 / (6 + 6\cos 2\theta) = 54 / (2 + \cos 2\theta)$$

Therefore, the sum of the infinite sequence is indeed equal to $54 / (2 + \cos 2\theta)$.

4 out of 4.

Find the values of θ which give the greatest value of the sum.

To find the values of θ that give the greatest value of the sum, we need to find the maximum value of the expression $54 / (2 + \cos 2\theta)$. Since $\cos 2\theta$ is always between -1 and 1 , the maximum value of this expression is obtained when $\cos 2\theta$ is equal to -1 . This occurs when $\theta = \pi/2 + k\pi$, where k is an integer. Therefore, the values of θ that give the greatest value of the sum are $\theta = \pi/2 + k\pi$, where k is an integer.

5 out of 6. Doesn't give the answers in the domain.

Total 49.

Level 5 [Level 6 is 54. 7 is 65+].