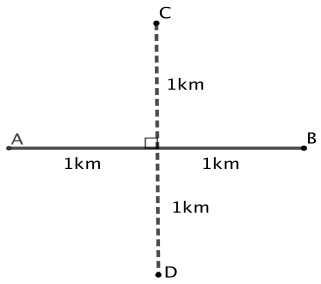
Avoiding a magical barrier.

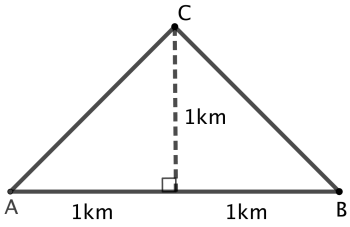
1. [Maximum mark: 33]

Each day you walk from Town A to Town B, a distance of 2km. Exactly 50% of the time there is no obstruction along this route. However, 50% of the time there is a magical barrier perpendicular to the route exactly half way between A and B, extending for 1km in both directions. This barrier is invisible and can only be sensed when you meet it.



Your task is to investigate the optimum strategy for minimizing your average journey.

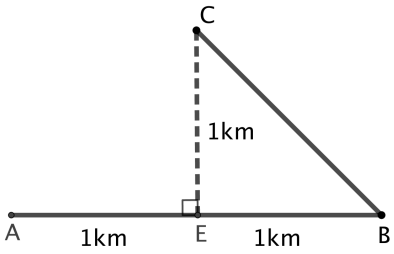
(a) Every day you set out directly on the route AC. You then walk in the line CB.



Find the distance travelled.

[2]

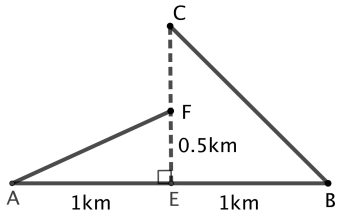
(b) Every day you set out directly on the route AB. If there is no barrier you continue on this straight line to B. If you meet the barrier you walk along the line EC and then in the straight line CB.



Find the average distance travelled.

[4]

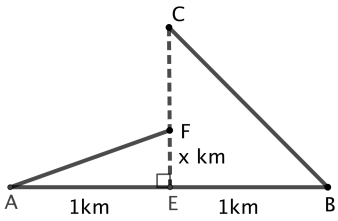
(c) Every day you set off on the line AF. F is 0.5 km from E along the perpendicular bisector of AB. If you meet the barrier you travel on the line FC before travelling in the line CB. If you don't meet the barrier you travel on the line FB.



Find the average distance travelled.

[4]

(d) Every day you set off on the line AF. F is x km from E along the perpendicular bisector of AB. If you meet the barrier you travel on the line FC before travelling in the line CB. If you don't meet the barrier you travel on the line FB.



(i) Show that the equation for the average distance travelled, y can be written as:

$$y = 1.5\sqrt{1+x^2} + \frac{1}{2}\left(1 - x + \sqrt{2}\right)$$

[3]

(ii) Use calculus to find the exact value of x, which minimizes the average distance, travelled.

[5]

(iii) Sketch a graph to verify your result graphically. What is the minimum average distance travelled?

[2]

(iv) The barrier now appears n % of the time. Show that the value of x, which minimizes the average distance, is given by:

$$x = \frac{n}{20\sqrt{(-n+100)}}$$

[8]

(v) For what values of n is the optimum strategy to head in a straight line from A to C?

[5]