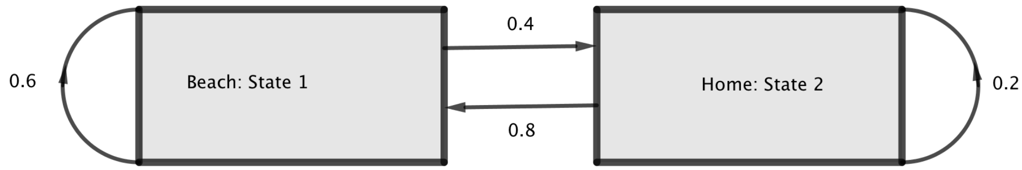
Intermathematics
Maths Resources for International Schools

Life's a Beach

1. [Maximum mark: 32]

Whilst on holiday you exist in two different states: State 1 is being on the beach and State 2 is relaxing at your beach home. If you are on the beach then the probability you are still on the beach after one hour is 0.6. If you are at home the probability you are still at home after one hour is 0.2. This is represented in the diagram below:



(a) Write down the transition matrix $\boldsymbol{M}$ for this Markov chain problem.

[2]

(ii) Find $\boldsymbol{M}^2$.

[2]

(iii) If someone starts on the beach, what is the probability they are at home after two hours?

[1]

(iv) If someone starts at home what is the probability that they are at home in 3 hours?

[2]

(b) Find the eigenvalues for matrix $\boldsymbol{M}$

[4]

(ii) Find the associated eigenvectors for matrix $\boldsymbol{M}$

[4]

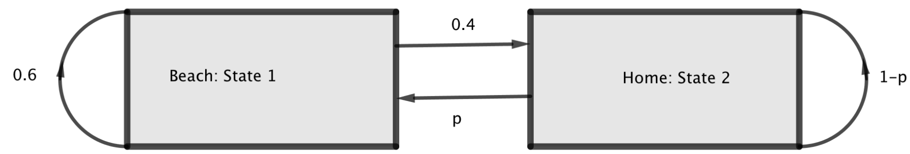
(iii) Show that:

$$\lim_{n\to\infty} \boldsymbol{M}^n = \begin{pmatrix} \frac{2}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{1}{3} \end{pmatrix}$$

Explain what your result means in the context of this scenario.

[5]

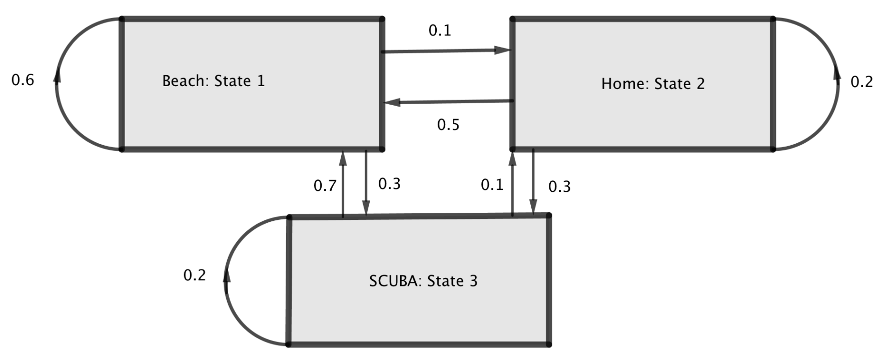
(c) You decide that you are spending too much time on the beach so you change your routine such that if you are at home the probability you are on the beach after one hour is p. This is represented below:



Find p such that for someone starting at home, the probability of being on the beach after 2 hours is 0.5. Give your answer in exact form.

[4]

(d) You now decide to also learn to SCUBA dive. There are now three states: Beach, Home and SCUBA. This is represented in the diagram below:



(i) Write down the transition matrix $\boldsymbol{M}$ for this Markov chain problem.

[2]

(ii) Initially there are 100 people on the beach, 200 people at home and 50 people SCUBA diving. If each person moves states with the same probabilities given above, how many people would we expect in each location after 2 hours?

[4]

(iii) Find a hypothesis for the likely long-term steady state of people at each location.

[2]