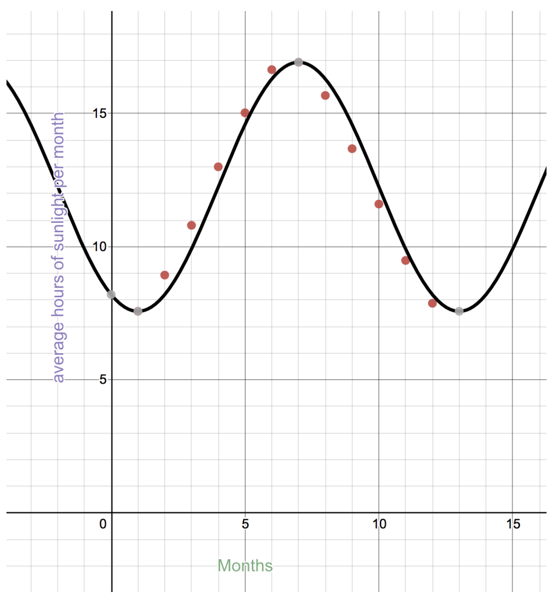
Mathematical Modelling Guide

A guide to help with the exploration:
Maths IB Standard Level and Higher Level
Applications and Interpretations
Analysis and Approaches

(For first examination in 2021).



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Introduction

I've written this guide to supplement the main Exploration Guide I put together. You should consult the main guide for guidance on choosing topics, an explanation of the marking criteria, common student mistakes and technology advice. In this guide I look at various modelling techniques. In many cases these are taught in textbooks simply using technology, whereas it is often desirable to demonstrate a greater understanding through non-calculator methods in your maths exploration. So, where possible I've included non-calculator techniques.

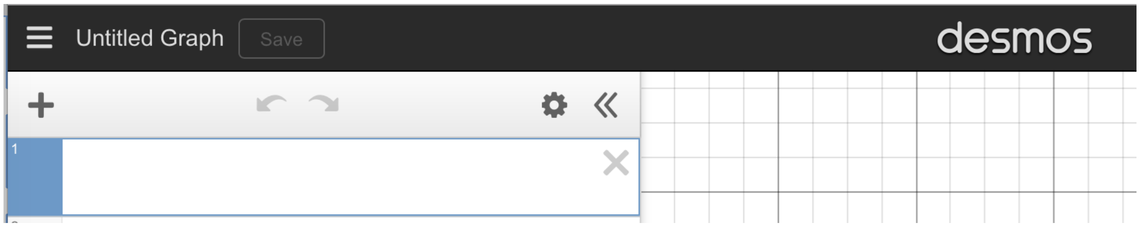
It's important to note that these methods are **not** intended to be exemplars - there are many different ways of explaining the following techniques and ideas, these are just my ideas! You should attempt to put your methods into your own words so that you can demonstrate a good personal understanding. The students who do best in their exploration consult from a variety of sources, collate the ideas and are therefore able to show a deep understanding.

If you do use this guide then it is essential that you correctly cite this source in your exploration - failure to cite sources correctly can lead to malpractice investigations by the IB, so make sure everything is done correctly.

The exploration is a great opportunity to apply your maths knowledge to an area of personal interest - so choose something you are passionate about, and enjoy it!

Desmos: Modelling

Desmos is a very powerful tool for modelling graphs. If you have a small number of data points you can add them in a table by clicking on the plus symbol and then selecting table.



If you have a large number of data points you can copy the relevant 2 columns from a spreadsheet program such as Excel and then paste them straight into this space and it will automatically convert this to a table.

x_1	y_1
0	0.6
1	0.7
2	1
3	1.8

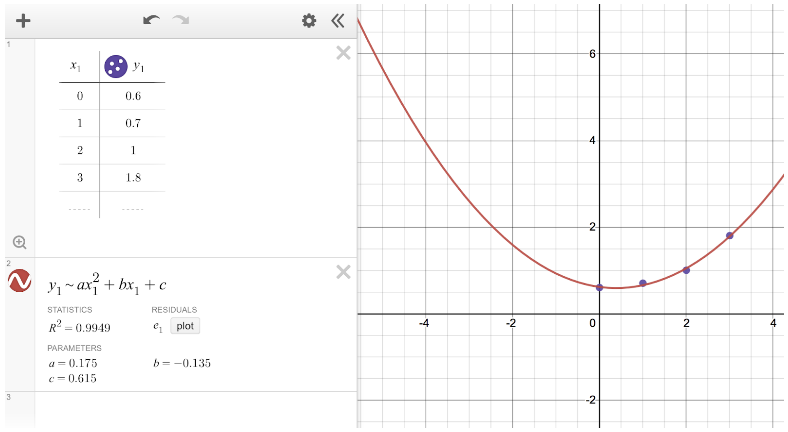
We can see that our x values are denoted as x_1 and our y values are denoted as y_1. This then allows us to specify a regression using values from this table. If we create another table we will see that our x values in this new are denoted as x_2 etc.

To create a quadratic regression line in Desmos I now can type the following general form of a quadratic equation:

$$y_1 \sim ax_1^{\,2} + bx_1 + c$$

This tells Desmos that I have 3 values to find: a, b, c, and to use the values from my table for x_1 and y_1. You will be able to type an underscore y_1 by pressing y then shift and the takeaway key (next to 0). You will be able to type the wiggly line by pressing shift and the key to the left of 1.

This then returns the following:



This tells me the values of a,b,c which best fit these points are:

a = 0.175
b = - 0.135
c = 0.615

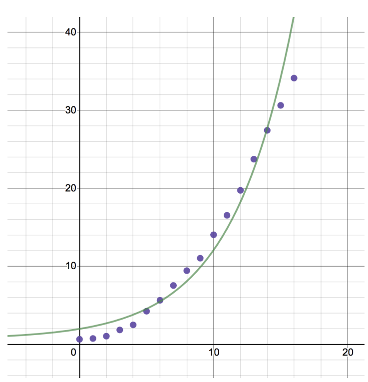
Which creates the following equation:

$$y = 0.175x^2 - 0.135x + 0.615$$

I can also see the R^2 statistic which measures the goodness of fit. R^2 close to 1 shows a very good fit - our R^2 value of 0.9949 shows that this curve fits the data very well.

Using the same idea I can then fit any curve using regression. For example an exponential regression would look like this:

$$y_1 \sim ae^{bx_1} + c$$



Summary of some different regression models on Desmos:

Polynomial

Linear: $y_1 \sim ax_1 + b$

Quadratic: $y_1 \sim ax_1{}^2 + bx_1 + c$

Cubic: $y_1 \sim ax_1{}^3 + bx_1{}^2 + cx_1 + d$

(Consider a higher order polynomial if there are more than 2 turning points).

Trigonometric

Sine curve: $y_1 \sim asin(b(x_1 + c)) + d$

Cosine curve: $y_1 \sim acos(b(x_1 + c)) + d$

(You might need to fix b if you get a graph which is too frequent).

Exponential

$$y_1 \sim ae^{bx_1} + c$$

(will fit both exponential growth and decay models)

Logistic

$$y_1 \sim \frac{a}{1+be^{-r(x_1-c)}}$$

(Will fit population growth models which start with high growth and gradually slow to a limit).

Ellipse and circles

$$\frac{(x_1-h)^2}{a^2} + \frac{(y_1-k)^2}{b^2} = 1$$

(The general equation of an ellipse - but will also fit circles).