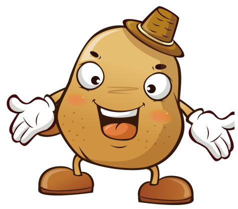
Who killed Mr. Potato? [36 marks]

1. *[Maximum marks: 12]*

Cooling rates are an essential tool in forensics in determining time of death. We can use differential equations to model the cooling of a body using Newton's Law of Cooling. The rate of change in the body temperature with respect to time is proportional to the difference in temperature between the body and the ambient room temperature.



In the following investigation we will determine when a potato was removed from an oven. If the ambient room temperature is 75 degrees Fahrenheit and T *is the temperature of the potato at time* t*, then we have:*

$$\frac{dT}{dt} = -k(T - 75)$$

a) *Show that the solution to this differential equation is:*

$$T - 75 = Ae^{-kt}$$

[4]

We arrive at a room at midday 12:00 to discover a potato on the kitchen counter. The ambient room temperature is 75 degrees Fahrenheit and we know the initial temperature of the potato before being taken out of the oven was 194 degrees Fahrenheit. We take the following measurements:

Time after 12:00 (t mins)	*Temperature of potato (T degrees Fahrenheit)*	*Difference from ambient room temperature* T_d *(T – 75)*
0	*133*	*58*
30	*116*	*41*
60	*104*	*29*
90	*98*	*23*
120	*91*	*16*

(b) *By using the measurements when* $t = 0$ *and* $t = 120$*, find* A *and* k*. Use your model to predict when the potato was first removed from the oven.*

[8]

***2.** [Maximum marks: 12]*

In this question we use an alternative method to estimate k.
If we take $T_d = T - 75$, then we have:

$$T_d = Be^{-kt}$$

(a) By finding the regression line of $ln(T_d)$ on t, find an estimate for k, B. State the correlation coefficient and comment on this value.

[8]

(b) Use your model to predict to the nearest minute when the potato was first removed from the oven. Compare your answer with (1c). Which method of finding k is likely to be more accurate?

[4]

***3.** [Maximum marks: 14]*

In this question we use a modified version of Newton's law of cooling introduced by Marshall and Hoare.
The modified equation includes an extra exponential term to account for an initial, more rapid cooling. We can write this as:

$$\frac{dT_d}{dt} + kT_d = 119ke^{-pt}$$

(a) Show that the solution to this differential equation is:

$$T_d = \frac{119k}{k-p}e^{-pt} + Ae^{-kt}$$

[7]

*(b) This model requires that we set $t = 0$ as the time when the potato was **first removed from the oven** (at this time $T_d = 119$). We are given that $k \approx 0.012$, $p \approx 9.5$. Find A and use your model to predict how many minutes it took for T_d to reach 58. Compare your 3 results.*

[7]