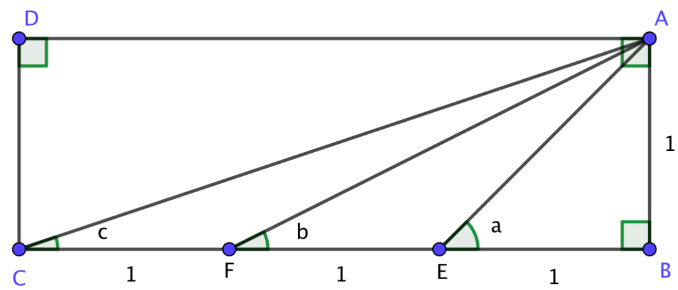
Quadruple Proof. [34 marks]

[This entire investigation is intended to be completed ***without*** *a calculator and hence non-calculator method should be shown throughout]*

1. *[Maximum marks: 8]*



The diagram above shows a rectangle ABCD. CF=FE=EB = 1. $a \leq \frac{\pi}{4}$ *and* $a > b > c$.

(a) *Find* $\sin(a)$, $\sin(b)$, $\sin(c)$.

[2]

(b) *Hence, by considering* $\sin(b + c)$ *prove that* $a + b + c = \frac{\pi}{2}$.

[6]

2. *[Maximum marks: 6]*

(a) *Find* $\tan(a)$, $\tan(b)$, $\tan(c)$.

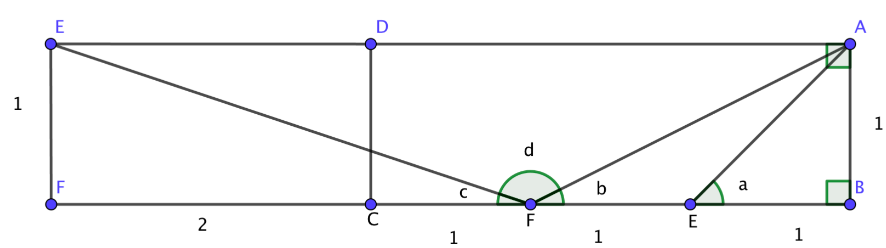
[1]

(b) *By letting* $b = \arctan(X)$, $c = \arctan(Y)$, *find an expression for* $\tan(b + c)$ *and hence prove that* $a + b + c = \frac{\pi}{2}$.

[5]

3. *[Maximum marks: 5]*

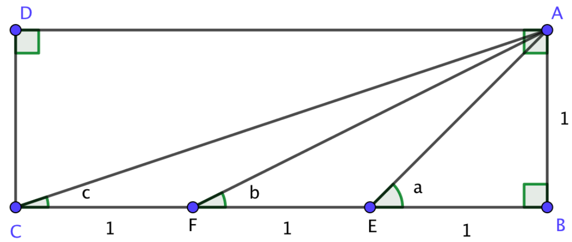
The diagram can be extended as follows:



Angle d is the angle AFE. By considering the triangle AFE, use the cosine rule to find angle d. Hence prove that $a + b + c = \frac{\pi}{2}$.

[5]

4. *[Maximum marks: 15]*
This proof should be attempted ***without*** *any trigonometry using a,b,c only.*



(a) *By taking A: (0,0), define E, F and C in terms of complex numbers in Cartesian form.*

[2]

(b) *Find the product ABC of the three complex numbers.*

[3]

(c) *Explain why the complex number E can also be written as* $\sqrt{2}e^{(-\pi+a)i}$

[2]

(d) *Find equivalent expressions for F and C.*

[2]

(e) *Hence find an alternative expression for the product ABC, in terms of a,b,c.*

[3]

(f) *Hence prove that* $a + b + c = \frac{\pi}{2}$.

[3]