

Optimization of area: an investigation [42 marks]

A farmer has 40m of fencing in every case. In this task you will explore the maximum area he can enclose.

1. [Maximum marks: 23]

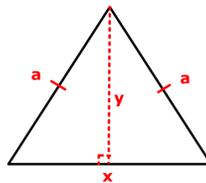
(a) What is the maximum area when the fence is in the shape of a circle? Leave your answer in terms of π .

[3]

(b) What is the maximum area when the fence is in the shape of a rectangle?

[4]

The farmer now makes a fence in the shape of an isosceles triangle.



(c) Find an equation for a in terms of x and y .

[3]

(d) Find an equation for y in terms of x only.

[3]

(e) Show that the area of the triangle can be written as:

$$A = \sqrt{5}\sqrt{20x^2 - x^3}$$

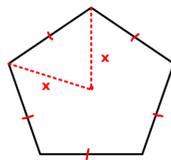
[5]

(f) Hence use **calculus** to find the value of x that maximizes the area and the area of the triangle. Comment on the value of x that you obtain.

[5]

2. [Maximum marks: 19]

(a) The farmer makes a fence in the shape of a regular pentagon.



Find an expression for x , the distance from the centre to a vertex and hence find the area of this pentagon.

[4]

- (b) Find an equation in terms of n for the area of an n sided regular polygon, $n \geq 3$.
[6]
- (c) Sketch a graph of the area of an n sided regular polygon (y -axis) versus n (x axis). Hence find the limit of the maximum area as n approaches infinity. Can you explain the significance of this result geometrically?
[3]
- (d) By making use of the substitution $u = \frac{1}{n}$ or otherwise, can you use L'Hopital's rule to find the limit as n approaches infinity?
[6]