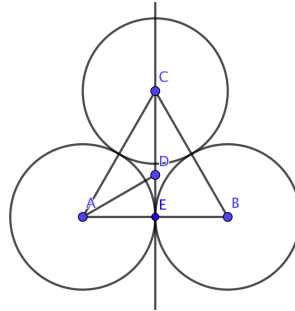


Circle packing density [31 marks]

1. [Maximum marks: 31]

In this question we investigate the density of circles in a given space.

Below we have 3 circles tangent to each other, each with radius 1. Point A has coordinates $(0,0)$. Point D is at the intersection of the three angle bisectors of the triangle.



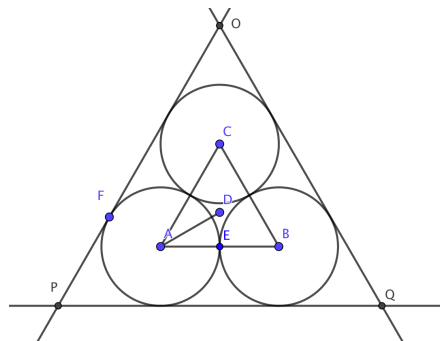
- (a) Find the coordinates of Point B, Point C and Point D [5]
- (b) Find the percentage of the triangle ABC that is not filled by the circles. [4]
- (c) The general equation of a circle with radius r and centered at (a,b) is given by:

$$(x - a)^2 + (y - b)^2 = r^2$$

Write down the equation of the three circles.

[2]

- (d) The tangents to the three circles make an equilateral triangle OPQ.



Show that the coordinate point F has coordinates $(\frac{-\sqrt{3}}{2}, \frac{1}{2})$.

[7]

- (e) *By first finding the equation of the tangent PQ, find the coordinates of P and Q,*
[7]
- (f) *Hence find the area of the triangle OPQ.*
[3]
- (g) *Find the percentage of the triangle OPQ that is not filled by the circles.
Comment on which triangle has a higher circle packing density.*
[3]