Using the Binomial Expansion for bounds of accuracy [29 marks]

1. *[Maximum marks: 8]*

(a) *Find the binomial expansion for* $\frac{1}{1+x}$ *for* $|x| < 1$. *Give your answer in terms of an infinite summation.*

[3]

(b) *Hence show that:*

$$ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots \; |x| < 1$$

[2]

(c) *By considering the approximation for ln(1+x), how many terms of the expansion are needed to find ln(1.1) correct to 4 decimal places?*

[3]

2. *[Maximum marks: 9]*

(a) *Find the full binomial expansion for* $\frac{1}{1+x^2}$ *for* $|x| < 1$ *and hence show that:*

$$arctan(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots \; |x| < 1$$

[4]

(b) *By considering x =*$\frac{\sqrt{3}}{3}$*, and using the approximation for arctan(x) up to* x^7 *term, find a lower bound for* π *to 3 decimal places.*

[3]

(c) *Find an upper bound for* π *to 3 decimal places that would be accurate to 2 decimal places.*

[2]

3. *[Maximum marks: 12]*

(a) *By considering:*

$$\int \frac{1}{\sqrt{1-x^2}}\, dx \; |x| < 1$$

Find a polynomial approximation for arcsin(x) up to and including the x^7 *term. State clearly the constant of integration.*

[5]

(b) *Find a polynomial approximation for arcos(x) up to and including the* x^7 *term.*

[3]

(c) *Use your answers to part (b) to find* $2(arccos(x) + arcsin(x))$

[2]

(d) *Use your answer to part (c) to explain why the following inequalities hold.*

$$-arccos(x) - arcsin(x) < arctan\ (x) < arccos(x) + arcsin(x)$$

[2]