Solving differential equations by separating variables

EXAMPLE 1

(a) Solve the differential equation $\frac{dy}{dx} = \frac{x^2}{y^2}$.

(b) Find the solution of this equation that satisfies the initial condition $y(0) = 2$.

$$\int dy \cdot y^2 = \int x^2\, dx$$

$$\frac{y^3}{3} = \frac{x^3}{3} + c$$

$$\frac{2^3}{3} = \frac{0^3}{3} + c$$

$$c = 8/3$$

$$y^3/3 = x^3/3 + 8/3 \quad \text{OR//} \quad y = \sqrt[3]{x^3 + 8}$$

V EXAMPLE 2 Solve the differential equation $\frac{dy}{dx} = \frac{6x^2}{2y + \cos y}$.

$$\int 2y + \cos y \; dy = \int 6x^2\, dx$$

$$\frac{2y^2}{2} + \sin y = \frac{6x^3}{3} + c$$

$$y^2 + \sin y = 2x^3 + c$$

Solve:

$$\frac{dy}{dx} = x^2 y$$

Write your answer in the form y = Ae^{f(x)} where A is a constant. If the curve passes through (0,1), find A.

$$\int \frac{dy}{y} = \int x^2 \, dx$$

$$\ln y = \frac{x^3}{3} + c$$

$$y = e^{x^3/3 + c}$$

$$y = A \cdot e^{x^3/3}$$

$$1 = A e^0$$

$$A = 1 \quad \therefore \quad y = e^{x^3/3}$$

Solve:

$$\frac{dI}{dt} = 15 - 3I \qquad I(0) = 0$$

$$\int \frac{dI}{15 - 3I} = \int dt$$

$$-\frac{1}{3} \ln(15 - 3I) = t + c$$

$$\ln (15 - 3I)^{-1/3} = t + c$$

$$(15 - 3I)^{-1/3} = e^{t + c}$$

$$(15 - 3I)^{-1/3} = A e^{t}$$

$$(15)^{-1/3} = A$$

$$\therefore (15 - 3I)^{-1/3} = 15^{-1/3} e^{t}$$

$$15 - 3I = 15 e^{-3t}$$

$$I = \frac{15 e^{-3t} - 15}{-3}$$

$$I = 5 - \cancel{3}5 e^{-3t}$$