EXAMPLE 12 Evaluate $\lim_{x \to 0} \dfrac{e^x - 1 - x}{x^2}$.

Using the expansion of e^x in the formula book

$$\lim_{x \to 0} \frac{\left(1 + \frac{x}{1} + \frac{x^2}{2!} + \frac{x^3}{3!} \cdots\right) - 1 - x}{x^2}$$

$$\lim_{x \to 0} \quad \frac{1}{2} + \frac{\frac{x^3}{3!}}{x^2} + \cdots = \frac{1}{2} + 0 \cdots = 1/2.$$

EXAMPLE 13 Find the first three nonzero terms in the Maclaurin series for (a) $e^x \sin x$

Using the expansion of e^x and sinx in the formula book

$$\left(1 + x + \frac{x^2}{2} + \frac{x^3}{6} \cdots\right)\left(x - \frac{x^3}{3!}\right)$$

$$= x + x^2 + \frac{x^3}{2} - \frac{x^3}{6}$$

$$= x + x^2 + \frac{x^3}{3}$$

5–12 Find the Maclaurin series for $f(x)$ ı

5. $f(x) = (1 - x)^{-2}$

Find the first 3 non zero terms. Can you write as a summation to infinity?

$f(0) = (1-0)^{-2} = 1$

$f'(x) = +2(1-x)^{-3}$ $\qquad f'(0) = 2(1)^{-3} = 2$

$f''(x) = +6(1-x)^{-4}$ $\qquad f''(0) = 6(1)^{-4} = 6$

$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0)$

$f(x) = 1 + 2x + 3x^2 + \ldots$

$\boxed{f(x) = \sum_{n=1}^{\infty} n x^{n-1}}$

6. $f(x) = \ln(1 + x)$

Find the first 3 non zero terms by differentiation. Can you write as a summation to infinity?

$f(0) = \ln(1) = 0$

$f'(x) = \frac{1}{1+x}$ $\qquad f'(0) = \frac{1}{1}$

$f''(x) = -(1+x)^{-2}$ $\qquad f''(0) = -1$

$f'''(x) = 2(1+x)^{-3}$ $\qquad f^{3}(0) = 2$

$\therefore\ f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f^{3}(0) + \ldots$

$= x + \frac{-1}{2}x^2 + \frac{2}{6}x^3 + \ldots$

$f(x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n} x^n$