Limits of functions 2

Name……………

V EXAMPLE 3 Evaluate

$$\lim_{x\to\infty}\frac{3x^2-x-2}{5x^2+4x+1}$$

$$= \lim_{x\to\infty}\frac{3-\frac{x}{x^2}-2/x^2}{5+\frac{4x}{x^2}+1/x^2}$$

$$= 3/5$$

EXAMPLE 11 Find $\lim_{x\to\infty}\dfrac{x^2+x}{3-x}$. $= -\infty$

$$= \lim_{x\to\infty}\frac{x+1}{3/x-1}$$

22. $\lim_{x\to\infty}\dfrac{x^2}{\sqrt{x^4+1}}$ $= \lim_{x\to\infty}\dfrac{x^2}{x^2\sqrt{1+1/x^4}} = \lim_{x\to\infty}\dfrac{1}{1+\frac{1}{x^4}} = 1$

$$\sqrt{x^4+1} = \sqrt{x^4(1+1/x^4)} = \sqrt{x^4}\sqrt{1+1/x^4}$$

25. $\lim_{x\to\infty}\left(\sqrt{9x^2+x}-3x\right)$

(Make into a fraction first)

$$\lim_{x\to\infty}\frac{\left(\sqrt{9x^2+x}-3x\right)\left(\sqrt{9x^2+x}+3x\right)}{\sqrt{9x^2+x}+3x}$$

$$= \lim_{x\to\infty}\frac{9x^2+x-9x}{\sqrt{9x^2+x}+3x} = \underline{\underline{\infty}}$$

$$\lim_{x\to\infty}\frac{9x^2-8x}{x\sqrt{9+1/x}+3x} = \lim_{x\to\infty}\frac{9x-8}{\sqrt{9+1/x}+3}$$

EXAMPLE 6 Evaluate $\lim_{x\to 2^+}\arctan\left(\dfrac{1}{x-2}\right)$.

(By considering the graph)

as $x\to 2^+$ $\lim_{x\to 2^+}\dfrac{1}{x-2} = \infty$

and $\lim_{u\to\infty}\arctan u = \underline{\underline{\pi/2}}$

EXAMPLE 11 Show that $\lim_{x \to 0} x^2 \sin \frac{1}{x} = 0$.

(Use the Squeeze Theorem)

$-1 \le \sin u \le 1$ $\quad \lim_{x \to 0^-} -x^2 = 0$ and $\lim_{x \to 0^+} x^2 = 0$

$-1 \le \sin \frac{1}{x} \le 1$

$-x^2 \le x^2 \sin \frac{1}{x} \le x^2$ $\quad \therefore \lim_{x \to 0} x^2 \sin \frac{1}{x} = 0$

57. (a) Use the Squeeze Theorem to evaluate $\lim_{x \to \infty} \frac{\sin x}{x}$.

$-\frac{1}{x} \le \frac{\sin x}{x} \le \frac{1}{x}$ $\quad \lim_{x \to \infty} \frac{1}{x} = \lim_{x \to \infty} -\frac{1}{x} = 0$

$\therefore \lim_{x \to \infty} \frac{\sin x}{x} = 0$

EXAMPLE 1 Find $\lim_{x \to 1} \frac{\ln x}{x - 1}$.

Use L'Hospital $\quad$ have $\frac{0}{0}$ $\therefore$ LH ✓ $\lim_{x \to 1} \frac{1/x}{1} = 1$

EXAMPLE 2 Calculate $\lim_{x \to \infty} \frac{e^x}{x^2}$. $\quad \frac{\infty}{\infty}$ $\therefore$ LH ✓

Use L'Hospital twice

$\lim_{x \to \infty} \frac{e^x}{2x} = \frac{\infty}{\infty}$ $\therefore$ LH ✓

$\lim_{x \to \infty} \frac{e^x}{2} = \infty$

EXAMPLE 3 Calculate $\lim_{x \to \infty} \frac{\ln x}{\sqrt[3]{x}}$.

Use L'Hospital once then simplify

$\frac{\infty}{\infty}$ $\therefore$ LH ✓

$\lim_{x \to \infty} \frac{\frac{1}{x}}{\frac{1}{3} x^{-2/3}} = 3x^{-1} \cdot x^{2/3} = \lim_{x \to \infty} 3x^{-1/3} = 0$

$\frac{0}{0}$ $\therefore$ LH ✓

EXAMPLE 4 Find $\lim_{x\to 0} \dfrac{\tan x - x}{x^3}$.

$\lim_{x\to 0} \dfrac{\sec^2 x - 1}{3x^2} = \frac{0}{0} \therefore$ LH ✓

Use L'Hospital 3 times

$\lim_{x\to 0} \dfrac{(\cos x)^{-2} - 1}{3x^2} = \lim_{x\to 0} \dfrac{-2(\cos x)^{-3}(-\sin x)}{6x} = \frac{0}{0} \therefore$ LH

$= \lim_{x\to 0} \dfrac{6(\cos x)^{-4}(-\sin x)(-\sin x) + -2(\cos x)^{-3}(-\cos x)}{6} = \frac{2}{6} = \underline{\underline{1/3}}$

EXAMPLE 6 Evaluate $\lim_{x\to 0^+} x \ln x$.

Rewrite as a fraction and use L'Hospital

$\lim_{x\to 0^+} \dfrac{\ln x}{1/x}$ $\quad \frac{\infty}{\infty} \therefore$ LH ✓ $\quad \lim_{x\to 0} \dfrac{1/x}{-x^{-2}} = \lim_{x\to 0} -x = 0$

EXAMPLE 7 Compute $\lim_{x\to(\pi/2)^-} (\sec x - \tan x)$.

Rewrite as a fraction and use L'Hospital

$\lim_{x\to \pi/2} \dfrac{1}{\cos x} - \dfrac{\sin x}{\cos x} = \lim_{x\to \frac{\pi}{2}} \dfrac{1 - \sin x}{\cos x} = \frac{0}{0} \therefore$ LH ✓

$= \lim_{x\to \pi/2} = \dfrac{-\cos x}{-\sin x} = \frac{0}{1} = 0$